

# **The Source of Alfvén waves that heat the solar corona**

Alexander Ruzmaikin

Jet Propulsion Laboratory, California Institute of Technology

Mitchell A. Berger

Department of Mathematics, University College London

Short title: THE SOURCE OF ALFVÉN WAVES...

**Abstract.** We suggest a source for high-frequency Alfvén waves invoked in coronal heating and acceleration of the solar wind. The source is associated with small-scale magnetic loops in the chromospheric network. The loops are energy loaded due to the topological complexity they have on emergence and acquire through footpoint twisting by photospheric motions. Random reconnections of these loops with flux tubes of the open or quasi-open field result in the release of a train of high-frequency Alfvén waves carrying and depositing their energy into the corona. The overall spectrum of the waves has the  $1/f$  form. The range of frequencies is confined between 1 and several hundred Hz.

## Introduction

The heating of the solar corona and acceleration of the solar wind have a common origin related to magnetic fields [Hollweg, 1986]. Models of these processes prescribe the source of the heating and its power; c.f. [Esser *et al.*, 1997]. The physical nature of the heating mechanism remains a problem, although a number of crucial insights have been made in the study of this problem. The energy transport into the corona and the pressure to drive the high-speed wind can be provided by Alfvén waves. Low-frequency Alfvén waves ( $\leq 10^{-2}$  Hz), excited by photospheric convective motions, can provide a necessary heating and acceleration beyond the sonic point ( $\geq 5R_{\odot}$ ) [Hollweg, 1986]. High-frequency Alfvén waves ( $\geq 1$  Hz), that dissipate by ion-cyclotron-resonance damping close to the Sun, can provide the heating of the low corona and initial acceleration of the solar wind [Axford and McKenzie, 1992; Marsch and Tu, 1997]. The origin of these waves is associated with small-scale magnetic activity, such as reconnections, in the chromospheric network.

Here we suggest a specific mechanism of generation of high-frequency Alfvén waves by the network magnetic field. The basic process is the build-up and release of energy in topologically complex magnetic configurations observed as tangled magnetic loops rising into the corona [Parker, 1990; Berger, 1994; Galsgard and Nordlund, 1996].

## Minimum Energy of Topologically Complex Magnetic Fields

The field with the minimum energy of all fields with a given normal component at the solar surface is the potential field. Any equilibrium field with a current has higher energy. The current induces topological complexity of the field, such as twist, writhe and linking of field lines usually described by a topological invariant called "magnetic helicity". The magnetic helicity, however, does not characterize all of the topological complexity of the field: there are an infinite number of high-order invariants

c.f. [Ruzmaikin and Akhmetiev, 1994]. A simple measure of complex magnetic line entanglement of general nature, called "crossing number", was introduced by *Freedman and He* [1991]. *Berger* [1993] derived a lower bound for the energy of braided magnetic fields as a function of the crossing number and extended the concept of the crossing number to continuous fields.

A braid is defined to be a collection of curves stretching between two parallel planes  $z = 0$  and  $z = L$  (Figure 1). A one-string braid is trivial. Two-string braids can simulate a twist of magnetic lines around each other. With three or more strings the braid can simulate topologically complicated configurations. We identify the curves with magnetic lines or thin magnetic flux tubes and their positions at the lower and upper plane with the positive and negative footpoints of the flux tubes. This represents magnetic loops in the solar atmosphere assuming that the positive footpoints are well separated from the negative ones. What will be lost in such representation is the curvature of loops. The complexity of a braid is measured by the number of times the strings cross each other as seen in a projection. Because the braids are three-dimensional, this crossing number depends on the viewing point. However, the crossing number averaged over the viewing angle (say, a polar angle in the plane  $z=0$ ) is angle independent [Berger, 1993]. The averaged crossing number is not in itself a topological invariant. It has, however, a positive minimum (called here  $K$ ) which, as well as the minimum energy  $E$ , is a topological invariant [Freedman and He, 1991].

Consider a magnetic flux tube of length  $L$  and radius  $R$ . To simulate solar conditions we assume that the axial field  $B$  is much stronger than the perpendicular field. Then the minimum state in the first approximation is independent of fluctuations in the axial field [van Ballegooijen, 1986] and is given by [Berger, 1993]

$$E = aK^2, \quad a \approx 9 \times 10^{-3} L^{-1} \Phi^2 \quad (1)$$

where  $\Phi = BR^2$  is the magnetic flux of the braid.

## Relaxation Along the Minimum State

Convection zone dynamics can add magnetic energy to flux tubes in two primary ways. First, the magnetic flux emerging to the solar surface may have a high level of topological complexity acquired from shear motions inside the convection zone. A detailed analysis of observations on five newly emerged bipolar regions showed that their magnetic structure was incompatible with a potential distribution and matched a helical distribution well [Leka *et al.*, 1997]. Second, the already emerged flux tubes can be further entangled due to random photospheric shear motions [Sturrock and Uchida, 1981; van Ballegoijen, 1986; Berger, 1994]. The magnetic structures can lose their energy and topological complexity because of reconnections inside the structures or with neighboring structures. Reconnections with open structures can produce jets [Feldman *et al.*, 1993; Shibata, 1997].

A magnetic configuration of a general type tends to relax to a minimum energy state; in a topologically trivial case to a potential field. The minimum magnetic energy in a non-trivial case depends on topological invariants of the configuration, which are not changing in the course of magnetic relaxation [Moffatt, 1990]. An example of such relaxation is a transition of a twisted flux tube into a writhed (coiled) tube, as happens to a telephone cord (for a detailed study of this transition see Ricca [1995]).

The dynamics of solar flux tubes is defined by the rate of energy input and energy loss. The velocity of photospheric motions  $V$ , twisting the flux tubes, is relatively small ( $\leq 1$  km/s) compared to the Alfvén speed (typically  $\geq 100$  km/s in the low corona). For twisting a thin flux tube however, the characteristic time  $R/V$  can be comparable with the characteristic magnetic relaxation time  $L/V_A$ . Here  $R \ll L$  are the flux tube radius and its length. In contrast, twisting of one flux tube around another can take a much longer time than the relaxation because the footpoints must travel a large distance to move about each other. The reconnection typically takes several Alfvén times  $R/V_A$ . Here we assume that the magnetic relaxation from an arbitrary state to the minimum

state is faster than any other process. The general case requires consideration of a more sophisticated kinetic of the three processes: magnetic relaxation, twisting and reconnection.

It follows that a newly emerged magnetic structure with energy  $E_0$  and topological complexity  $K_0$  will rapidly move to the minimum state, releasing the excess energy in the form of Alfvén waves (Figure 2). If the crossing number of the structure is high enough then reconnections will move the configuration down the minimum curve releasing the energy in small portions. According to estimates by *Berger* [1994], the reconnections can provide a significant part of the heating requirement for closed loops when the ratio between the transverse and axial components of the field in the braid,  $B_t/B$ , exceeds 0.3, which corresponds to about a  $30^\circ$  angle between the directions of neighboring flux tubes. If the transverse field is smaller there is enough time for random convective motions to entangle the braid, thus increasing the crossing number and moving the structure up along the minimum state. This leads to a stationary situation in which the rate of magnetic energy input through the random motions is balanced by the rate of energy release through the reconnections. The power (per unit area) released due to the moves along the minimum state can be estimated from Eq. (1):  $P = (2a/\pi R^2)KdK/dt$ . It has been shown [Berger, 1994] that for a random-walk with stepsize  $\lambda$ :  $dK/dt = \epsilon V/\lambda$ , where  $V$  is a root-mean-square velocity of random motions and  $\epsilon \approx 0.5$ . The crossing number corresponding to the level of effective reconnection,  $B_t/B = \mu \approx 0.3$ , is  $K \approx 2\mu L/R$ . This results in

$$P \approx 0.1\epsilon\mu VB^2 \frac{R}{\lambda} \quad (2)$$

Although this stationary situation is probable, it might not be achieved for every flux tube due to the finite time of its existence on the Sun. If, however, the flux tubes emerge well twisted already, the transition to stationarity through reconnections is fast. Flux tubes which emerge already carrying topological complexity can also release

their energy into the Alfvén waves (see next section) before the stationary situation is achieved. In this case the power is determined by the crossing number acquired inside the convection zone. In the convection zone, as known at least from the equipartition argument, the velocity of motions and the Alfvén speed are of the same order. Hence, the stationary situation, i.e. the balance between increase and decrease of topological complexity, can easily be achieved and sustained. This results in the same estimate of the power as given in Eq. (2).

## Reconnection of Braided Structures with Open Flux Tubes

Following *Axford and McKenzie* [1992] we associate the configurations discussed above with closed magnetic loops in the chromospheric network. The size of these loops is of the order  $L = 3 \times 10^4$  km, the field is 5-10 G on average, and the Alfvén speed is of the order  $V_A = 10^3$  km/s. We expect, however, that the magnetic field inside the braided flux tubes is higher in proportion to the filling factor of the flux tubes on the photosphere. The chromospheric network, distributed all over the solar surface, has a different environment in different solar regions. In the fast wind regions, i.e. coronal holes, loops neighbor open magnetic lines. In the slow wind regions they neighbor large closed magnetic loops extending into the solar corona. Substituting the numerical values,  $V = 1$  km/s,  $B = 30$  G,  $R/\lambda = 1 - 2$ , into Eq. (2) gives an estimate  $(1 - 3)10^6$  erg/cm<sup>2</sup>s. This power has the same order of magnitude as the power needed for the heating the inner corona and the initial acceleration of the solar wind [*Marsch and Tu*, 1997].

We assume that the small-scale chromospheric magnetic loops evolve as described in the above section (accumulation of topological complexity balanced by reconnection). These reconnections can be identified with the "nanoflares" envisaged by *Parker* [1990]. More dramatic, however, is the other type of reconnection: From time to time, randomly, the topologically complex flux tube meets a flux tube of the open field of

opposite direction (or part of a large-scale loop) and reconnects with it releasing a train of Alfvén waves up into the corona (Figure 3). Because these waves arise from a twisted configuration, they are circularly polarized. The minimum wavelength can be estimated as  $L/K$ . The corresponding upper-bound frequency  $f = V_A K/L \approx 2\mu V_A/R$  is basically defined by the Alfvén speed and the typical radius of the flux tubes. Unfortunately, the radius of flux tubes is unknown because it is below the best spatial resolution (0.2 arcsec). We assume that the radius is of the order of magnitude of a thickness of a skin-layer determined by plasma resistivity, i.e. it is as small as 1-10 km. This gives an estimate of the upper-bound frequency as several hundred Hz. The lower-bound frequency is about  $V_A/L$ . The estimated range of frequencies is in agreement with the range suggested on observational grounds [Marsch and Tu, 1997].

Marsch and Tu [1997] assumed a  $1/f$  spectral distribution of waves, well justified by the spectrum of Alfvén waves observed in the solar wind. The origin of this spectrum can be understood as follows. Reconnections of energy-loaded flux tubes with open flux tubes are random events. Let us assume that the spectrum of an event has a simple Lorentz form  $e(f) \propto \tau/(1 + f^2\tau^2)$  with one parameter  $\tau$  – a characteristic time for this process- considered to be a random variable taking different values at different sites of the solar surface. The sampling of events coming from different sites is thus equivalent to averaging over this random variable. The resulting spectrum is given by  $S(f) = \int e(f, \tau)\rho(\tau)d\tau$ , where  $\rho(\tau)$  is the probability distribution function of  $\tau$  which can be approximated by a simple  $1/\tau$  law [Matthaeus and Goldstein, 1986; Ruzmaikin, 1996]. The scale-invariant weight  $\rho(\tau)d\tau \propto d\tau/\tau$  in the integral immediately results in the  $1/f$  spectrum.

## Conclusions

The suggested source of generation of high-frequency Alfvén waves fills a crucial gap in our understanding of coronal heating and solar wind acceleration. The mechanism



is based on a number of previously developed ideas, such as twisting of flux tubes by convective motions, emergence of topologically non-trivial magnetic fields, and reconnections of closed flux tubes with open ones. We have developed here the concept of the minimum state for solar flux tubes, which brings these ideas together. Another innovative step is the relation of topological changes in the flux tubes to energy release in the form of high-frequency Alfvén waves.

**Acknowledgment.** This research was conducted in part at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautic and Space Administration.

## References

- Axford, W. I. and J. McKenzie, The origin of high-speed solar wind streams, in *Solar Wind Seven*, edited by E. Marsch, R. Schwenn, Pergamon Press, pp.1–5, 1992.
- Berger M. A., Energy-crossing number relations for braided magnetic fields, *Phys. Rev. Lett.*, **70**, 705–708, 1993.
- Berger M. A., *Space Science Reviews*, **68**, 3, 1994
- Galsgard, K., and A. Nordlund, Heating and activity of the solar corona, *J. Geophys. Res.*, **101**, 13,445–13,460, 1996.
- Esser, R., S. Habbal, W. Coles, and J. Hollweg, Hot protons in the inner corona and their effect on the flow properties of the solar wind, *J. Geophys. Res.* **102**, 7063–7074, 1997.
- Feldman, W., J. Gosling, D. McComas, and J. Phillips, Evidence for ion jets in the high-speed solar wind, *J. Geophys. Res.* **98**, 5593–5605, 1993.
- Freedman, M. and He, Z. H., Divergence-free fields: energy and asymptotic crossing number, *Ann. of Math.*, **134**, 189–229, 1991.
- Hollweg, J. V., Transitional region, corona, and solar wind in coronal holes, *J. Geophys. Res.* **91**, 4111, 1986.
- Leka, K. D., R. Canfield, and A. N. McClymont, Evidence for current-carrying emerging flux, *Astrophys. J.*, **462**, 547–560, 1996.
- Marsch, E. and C.-Y. Tu, The effects of high-frequency Alfvén waves on coronal heating and the solar wind acceleration, *Astron. & Astrophys.*, **319**, L17-L20, 1997.
- Matthaeus, W. H., and M. L. Goldstein, Low-frequency 1/f noise in the interplanetary magnetic field, *Phys. Rev. Lett.*, **57**, 495-498, 1986.
- Moffatt, K., The energy spectrum of knots and links, *Nature*, **347**, 367–369, 1990.
- Parker, E. N., *Adv. Space Sci.*, **10**, 17, 1990.
- Sturrock, P. and Y. Uchida, Coronal heating by stochastic magnetic pumping. *Astrophys. J.*, **246**, 331–336, 1981.

Ricca, R., The energy spectrum of a twisted string under elastic relaxation, *J. Phys. A: Math. Gen.* 28, 2335–2352, 1995.

Ruzmaikin, A.

and

N. Akhmetiev, Topological invariants of magnetic fields , and effect of reconnections, *Phys. Plasmas*, 1, 331–337, 1994.

Ruzmaikin, A., B. Goldstein, E. J. Smith, and A. Balogh, On the Origin of the 1/f Spectrum in the Solar Wind, in *Solar Wind Seven*, edited by D. Winterhalter et al., pp.225–229, AIP Press, 1996.

Shibata, K., Rapidly time variable phenomena: jets, explosive events, and flares, in *The Corona and Solar Wind Near Minimum Activity*, ESA SP-404, edited by O. Kjeldseth-Moe, pp. 103–112, 1997.

van Ballegooijen, A. A., Cascade of magnetic energy as a mechanism of coronal heating, *Astrophys. J.*, 311, 1001–1014, 1986.

---

A. Ruzmaikin, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109, USA. (e-mail aruzmaik@jpl.nasa.gov)

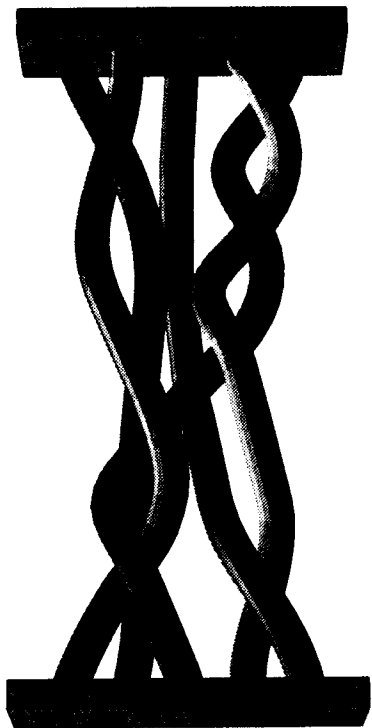
M. Berger, Department of Mathematics, University College London, London, England, WC1E 6BT. (e-mail mberger@math.ucl.ac.uk)

Received January, 1998; revised , 1998; accepted , 1998.

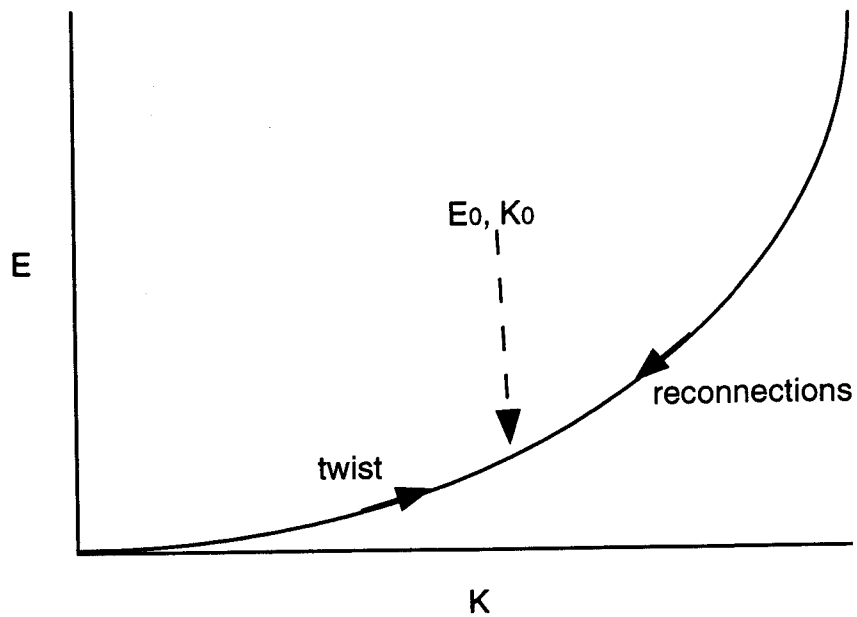
**Figure 1.** A topologically complicated magnetic loop configuration can be represented by a braid. The figure shows an example of three flux tubes braided with 12 crossings.

**Figure 2.** A flux tube emerging with arbitrary values of  $E_0$  and  $K_0$  evolves fast (in an Alfvén time) to the minimum state defined by the magnetic energy and crossing number. Reconnections destroy crossings and thus move the configuration down along the minimum state curve. Random photospheric motions increase topological complexity and thus move the configuration up along the curve.

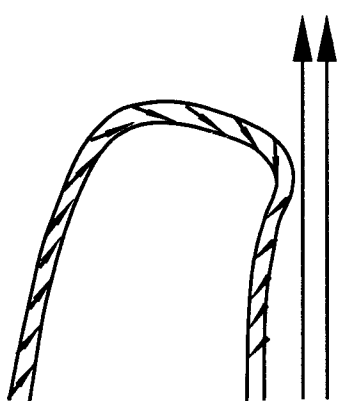
**Figure 3.** The reconnection of a topologically complex closed magnetic loop (a) with an open flux tube releases a train of circularly polarized Alfvén waves into the solar corona. The closed loop formed in this process continues to be a source of the “nanoflare” activity.



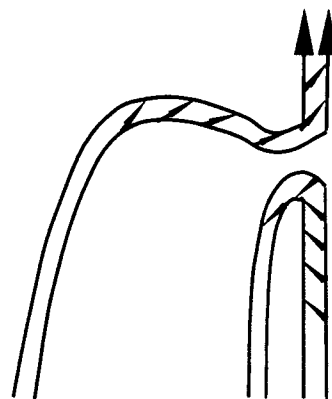
*fig. 1.*



*fig. 2*



a



b

*fig. 3*